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Influence of Pipeline Depth of Cover on Failure Frequencies due to External Interference

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Draft 04	Simplification and restructuring following internal review. Results based on full dataset (Case 2).	Mike Acton

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Distribution

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Executive Summary

The predicted failure frequencies for buried pipelines due to external (third party) interference damage are a function of both the probability that a pipeline will fail given that an accidental impact has occurred (which in turn depends on the severity of the damage and the ability of the pipeline to withstand that damage) and the frequency with which such impacts are expected (the “hit rate”). The average hit rate for pipelines (expressed as hits per km per year) is obtained by dividing the total number of recorded hits for a pipeline population by the corresponding pipeline exposure (in km years).

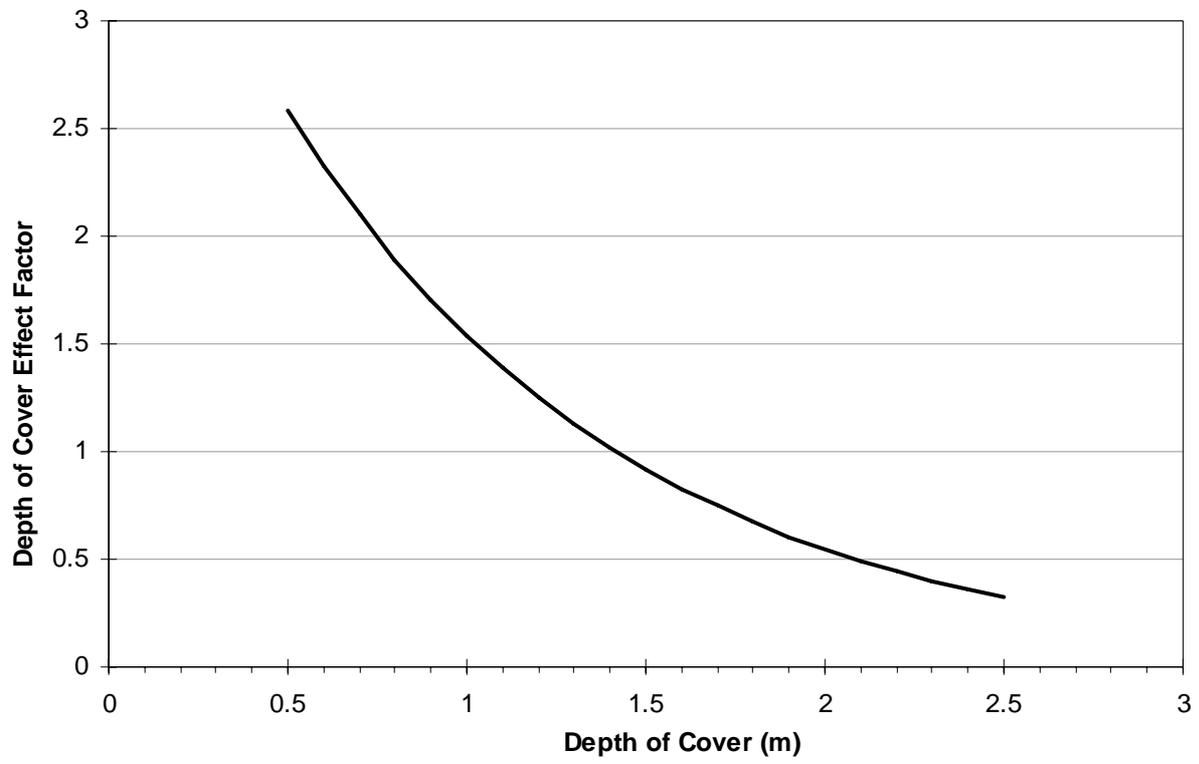
However, the depth of cover at which a pipeline is laid has a significant effect on the predicted failure frequency of the pipeline because a greater depth of cover reduces the frequency at which third party activities reach sufficient depth to hit the pipeline. Conversely, a reduction in the pipeline depth of cover increases the predicted hit rate and hence failure frequency. The principle that increased depth of cover is an important protective measure for buried pipelines is widely accepted in pipeline codes, such as IGEM/TD/1 and PD 8010 Part 1, which specify a minimum depth of cover required in order to protect the pipeline from third party activities.

The purpose of the present study, undertaken for the UK Onshore Pipeline operators Association (UKOPA), was to derive a relationship between the hit rate and the depth of cover of pipelines in order that factors may be applied to the average hit rate (and hence the predicted failure frequency) to take account of the actual depth of cover in pipeline risk assessments (following the methodology described in IGEM/TD/2 and PD 8010 Part 3, for example).

The effect of depth of cover on the failure frequency of a pipeline has been studied in previous work (most recently reported in 2004), which derived relationships between the failure frequency for external interference and the depth of cover. The approach in all of these studies used the depth of cover measured in reported corrosion incidents to estimate the depth of cover distribution for all pipelines in the database (based on the assumption that corrosion is independent of depth of cover). The work reported herein updates and extends the 2004 study by applying the same approach to the analysis of data from the UKOPA database, which includes information from a wider range of UK Major Accident Hazard Pipelines (and not only natural gas transmission pipelines owned by Transco/National Grid as considered previously).

It is important to appreciate that the relationship derived is the variation in the hit rate with depth of cover and not a measure of the absolute value of the hit rate. For example, a direct comparison of the absolute hit rates would show a substantial difference between Rural and Suburban pipelines, because the historical hit rate for Suburban area pipelines is considerably higher than the hit rate for Rural area pipelines. For the UKOPA dataset analysed in the study, the hit rate for Rural pipelines was calculated to be 0.37 per 1000 km.years and 1.5 per 1000 km.years for Suburban pipelines (higher by a factor of approximately 4). The average depth of cover estimated for the pipelines in the dataset was approximately 1.4m (the difference in average depth of cover between Suburban and Rural pipelines was small and is neglected here).

The relationship derived is presented below and shows the variation in hit rate with depth of cover (between 0.5m and 2.5m), where a factor of 1 corresponds to the average depth of cover estimated for the UKOPA pipelines. The “Depth of Cover Effect Factor” read from this figure corresponding to the actual depth of cover should be applied to the applicable hit rate in order to estimate the external interference hit rate for a particular pipeline.



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1 Introduction

The predicted failure frequencies for buried pipelines due to external (third party) interference damage are a function of both the probability that a pipeline will fail given that an accidental impact has occurred (which in turn depends on the severity of the damage and the ability of the pipeline to withstand that damage) and the frequency with which such impacts are expected (the “hit rate”). The average hit rate for pipelines (expressed as hits per km per year) is obtained by dividing the total number of recorded hits for a pipeline population by the corresponding pipeline exposure (in km years).

However, the depth of cover at which a pipeline is laid has a significant effect on the predicted failure frequency of the pipeline because a greater depth of cover reduces the frequency at which third party activities reach sufficient depth to hit the pipeline. Conversely, a reduction in the pipeline depth of cover increases the predicted hit rate and hence failure frequency. The principle that increased depth of cover is an important protective measure for buried pipelines is widely accepted in pipeline codes, such as IGEM/TD/1 [1] and PD 8010 Part 1 [2], which specify a minimum depth of cover required in order to protect the pipeline from third party activities.

The purpose of the present study, undertaken for the UK Onshore Pipeline operators Association (UKOPA), was to derive a relationship between the hit rate and the depth of cover of pipelines in order that factors may be applied to the average hit rate (and hence the predicted failure frequency) to take account of the actual depth of cover in pipeline risk assessments (following the methodology described in IGEM/TD/2 and PD 8010 Part 3 for example).

FFREQ is a well-established mathematical model for the prediction of pipeline failure frequencies due to third party interference originally developed in the 1980's [3]. FFREQ is currently the methodology recommended by UKOPA for predicting failure frequencies for buried pipelines due to third party interference damage. It combines historical data on the frequency and severity of damage (using Weibull distributions for the length and depth of gouges and gouges in dents) with a structural model that determines the severity of damage required to cause failure of the pipeline in question. This method allows the influence of the main pipeline-specific parameters (nominal pipeline diameter, pressure, wall thickness, material grade and toughness) on failure probability given a hit to be quantified. FFREQ divides damage into two types, gouges and dent/gouges and calculates the failure frequency for each type independently, then combines them to give the overall failure frequency

The effect of depth of cover on the failure frequency of a pipeline has been studied in previous work, which derived relationships between the failure frequency for external interference and the depth of cover. The approach in all of these studies used the depth of cover measured in reported corrosion incidents to estimate the depth of cover distribution for all pipelines in the database (based on the assumption that corrosion is independent of depth of cover). The following list describes (in chronological order) the previous work that has been carried out to derive depth of cover factors:

- In initial work, the effect of depth of cover was quantified using empirical methods and the results used to define the factors that should be applied to failure frequency predictions for external interference made using the FFREQ model to account for variations in the pipeline depth of cover.
- The effect of depth of cover on external interference failure frequencies was later investigated using incident data obtained from the Transco Fault Database (TFDB) which contained data for the period 1970-87 [4].

- The most recent work, reported in 2004 [5], used updated data from the Transco Fault Database (TFDB) taking account of additional data available since 1987.

The work reported herein updates and extends the previous 2004 study [5], by basing the analysis on data obtained from the UKOPA database [6], which includes information from a range of UK Major Accident Hazard Pipelines and not only natural gas transmission pipelines owned by Transco/National Grid.

2 The Relationship between the Hit Rate and the Depth of Cover

2.1 Methodology

The analysis presented here follows the same approach applied in the previous study [5]. The approach requires the interference hit frequency to be calculated as a function of depth of cover. The difficulty in this approach is that it is not possible to derive system exposure data (the distribution of pipelines at specific depth of covers) as a function of depth of cover, as the exposure data is insufficiently detailed. In order to overcome this, the exposure data (in terms of the proportion of the system at different depths of cover) is estimated from data on corrosion incidents, which are assumed to be independent of depth of cover.

Data on all corrosion incidents (i.e. all records of corrosion faults identified from inspection activities plus gas release incidents caused by corrosion) were obtained from the UKOPA fault database [7]. Incidents occurring at Above Ground Installations (AGIs) were excluded since the models that would be subject to the findings of this work are only used to calculate failure frequencies on buried cross-country transmission pipelines.

The data was used to produce a plot of the proportion of corrosion incidents as a function of depth of cover. Since it is assumed that corrosion incidents are independent of depth of cover, the distribution of corrosion incidents is assumed to approximate the distribution of the system exposure. Only external corrosion records are used when determining the depth of cover distribution. This is because internal corrosion incidents are influenced by the fluid composition, whereas it is assumed that external corrosion is independent of the fluid being transported. Data on external interference incidents (both incidents with no product loss and those with product loss) were then obtained to determine the proportion of interference incidents as a function of depth of cover.

By calculating the ratio of interference incidents to corrosion incidents for different depth of cover bands, it is thus possible to derive the relationship between pipeline depth of cover and the interference incident frequency ("hit rate"), based on a methodology originally developed by Gasunie [8]. The statistical method is described in Appendix B. Factors can then be derived to apply to external failure frequency values for different depth of cover values.

2.2 Determining the Depth of Cover Distribution

A plot of the proportion of external corrosion incidents as a function of depth of cover is shown in Figure 1 for all area types (i.e. both Rural and Suburban areas). The depth of cover presented in the figure is the centre of the depth of cover band that has a width of 0.2m. Since it is assumed that corrosion incidents are independent of depth of cover, the distribution of corrosion incidents is assumed to approximate to the distribution of the system exposure.

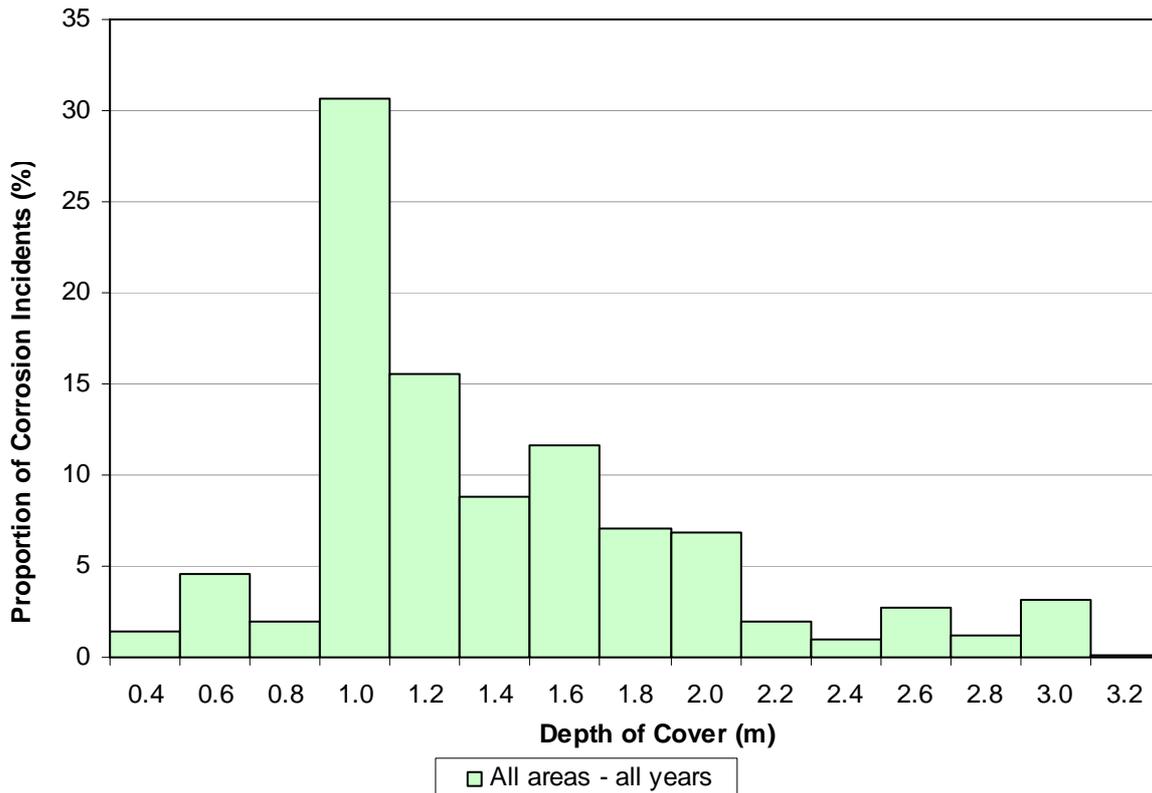


Figure 1: Corrosion incidents as a function of pipeline depth of cover

Based on analysis of this data and the assumption that external corrosion incidents are independent of pipeline depth, the average depth of cover was calculated for the system exposure to be 1.4m (the difference in average depth of cover between Suburban and Rural pipelines was small and is neglected here).

2.3 Interference Incident Distribution

A plot of the proportion of external interference incidents as a function of depth of cover is shown in Figure 2 for all area types. This represents the distribution of the third party hit rate with depth of cover and excludes damage that occurred during the original construction of the pipeline. The total number of interference records remaining for analysis from the period 1962 to 2010 was 399. The depth of cover presented in the figure is the centre of the depth of cover band that has a width of 0.2m.

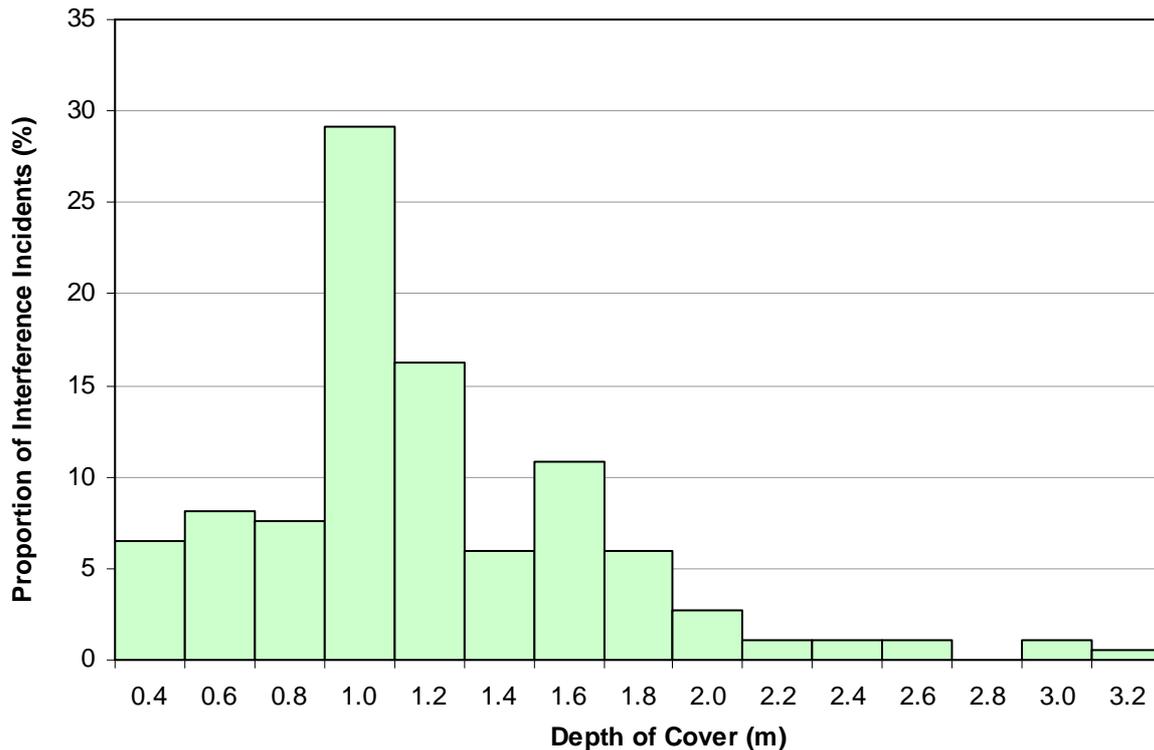


Figure 2: External interference incidents as a function of pipeline depth of cover

2.4 Correlating the Hit Rate with Depth of Cover

By calculating the ratio of third party hit rate to corrosion incidents for different depth of cover bands, it is possible to examine the effect of depth of cover on the third party hit rate. This is because this ratio is assumed to be proportional to the interference incident frequency.

The resulting fit has the following form,

$$f(d) = e^{Ad+B} \tag{Equation 1}$$

where A and B are constants, d is the depth of cover, and f is the natural log of the ratio of the proportion of interference incidents to the proportion of corrosion incidents.

Due to the scarcity of data at the extremes of the distributions, the depth of cover range used in the analysis was from 0.5m to 2.5m. As described in Appendix A, a sensitivity study was performed to investigate the effect of using either all of the available data in the UKOPA database or data from the past 20 years only (i.e. 1990 to 2010). In addition, the effect of increasing the widths of the bands at the extremes in order to provide a more representative spread of data across the bands for analysis.

Following the sensitivity study, the data analysis presented here was based on the full dataset, with broader depth of cover bands at the extremes of the distribution to give the following ranges:

Table 1 Depth of cover bands

ID	Min	Max
1	0.5	< 0.8
2	0.8	< 1.1
3	1.1	< 1.3
4	1.3	< 1.5
5	1.5	< 1.7
6	1.7	< 2.0
7	2.0	< 2.5

A plot of the natural log of the ratio of third party hit rates to corrosion incidents, as a function of depth of cover, for all area types and all years is shown in Figure 3. For positive values this indicates a greater proportion of interference incidents than corrosion incidents in the same depth of cover band. A least squares linear fit to this data has been derived as detailed in [8] (and summarised in Appendix B) and it can be shown with 95% confidence that the correlation between the data is statistically significant. The equation for the fit to the entire dataset (1962 – 2010) is:

$$f(d) = e^{-1.03d+1.35} \tag{Equation 2}$$

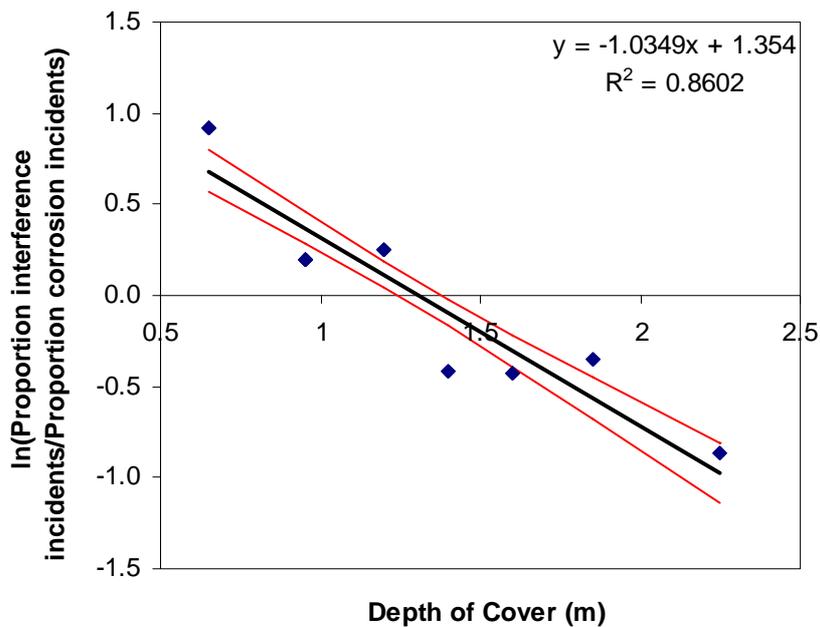


Figure 3: Natural log of the ratio of third party hit rates to corrosion incidents as a function of depth of cover (1962 – 2010)

2.5 Derivation of Factors for the Relationship between Hit Rate and Depth of Cover

It has been shown that there is a statistically significant correlation between the third party hit rate and the depth of cover. It is now possible to derive suitable factors, which represent the relationship between the hit rate and the depth of cover, to apply to failure frequency predictions.

The depth of cover factor can be set to be unity at a specific depth of cover, D , such that the depth of cover factor, F , takes the following form:

$$F(d) = \frac{e^{-Ad+B}}{e^{-AD+B}} \tag{Equation 3}$$

where D is a constant that is the depth of cover at which the equation is normalised (by definition the denominator will be constant).

The resulting curve showing the variation in hit rate with the depth of cover (the “Depth of Cover Effect Factor”) normalised at the average depth of cover for the pipeline exposure (i.e. $D = 1.4\text{m}$) is shown in Figure 4.

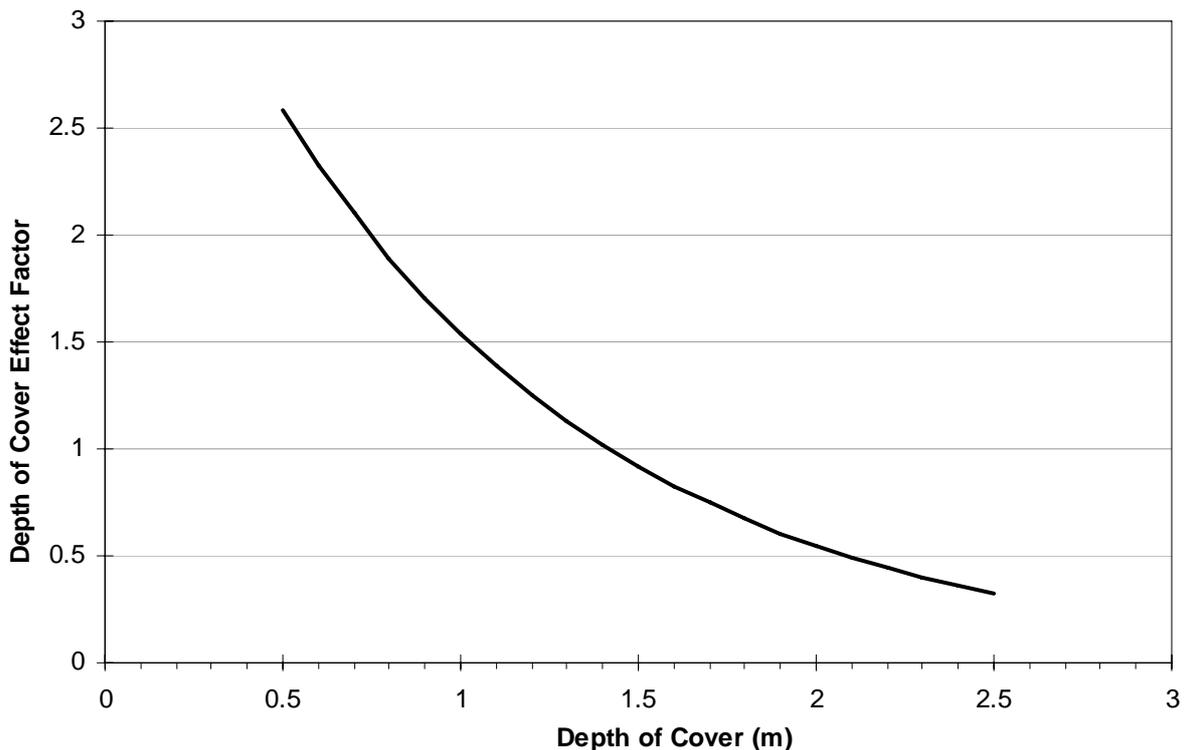


Figure 4: Depth of cover effect factor (normalised at mean depth of cover 1.4m)

2.6 Hit Rates

In order to apply the relationship in Figure 4, it is necessary to know the baseline hit rate corresponding to an effect factor of 1. For the full UKOPA dataset used in the analysis (1962 – 2010), hit rates were

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calculated separately for Rural and Suburban area pipelines by dividing the number of hits recorded in each area type by the system exposure of each type taken from the most recent UKOPA report [6].

For Rural area pipelines, the number of external interference damage incidents is recorded as 265 and corresponds to a system exposure of Rural pipelines of 711,658 km.year. Hence, the average hit rate for Rural pipelines is calculated to be 3.72×10^{-4} per km.year.

Similarly, for Suburban area pipelines, the number of external interference damage incidents is recorded as 111¹ and corresponds to a system exposure of Suburban pipelines of 72,722 km.year. Hence, the average hit rate for Suburban pipelines is calculated to be 1.53×10^{-3} per km.year.

For a given pipeline with a specified depth of cover, the appropriate average hit rate is then multiplied by the Depth of Cover Effect Factor from Figure 4 to estimate the hit rate for that pipeline.

¹ Note that the sum of the Rural and Suburban area external interference damage incidents is less than the total of 399 used for the depth of cover analysis because a small number of incidents are recorded on pipelines with a different area classification.

3 Discussion

3.1 Sensitivity Analysis

A number of sensitivity studies were performed to investigate the sensitivity of the relationship to the method of analysis and the dataset used. In particular, sensitivity studies were carried out to investigate the possibility of using data from the previous 20 years only (vs the full dataset) and the effect of broadening the width of the depth of cover bands at the extremes of the distributions, due to the sparsity of the data. The results of these sensitivity studies are presented in Appendix A. It was found that limiting the dataset to the past 20 years only (i.e. 1990 – 2010) significantly reduced the confidence on the fits, due to the limited data available for statistical analysis (i.e. 108 records from the previous 20 years compared with 399 records for the full dataset). Similarly, the fit to the data was improved by apply broader depth of cover bands at the extremes of the distributions, in order to provide a more uniform distribution of data points in each band

Following these investigations, the full dataset was selected as the basis of the relationship proposed here. In addition, broader bands were used at the extremes of the distributions, as presented in Section 2.4.

3.2 Comparison with UKOPA FFREQ

Figure 5 presents the derived depth of cover factor curve compared against the curve currently used in the UKOPA FFREQ model. The FFREQ relationship implies a slightly shallower average depth of cover (approximately 1.3m).

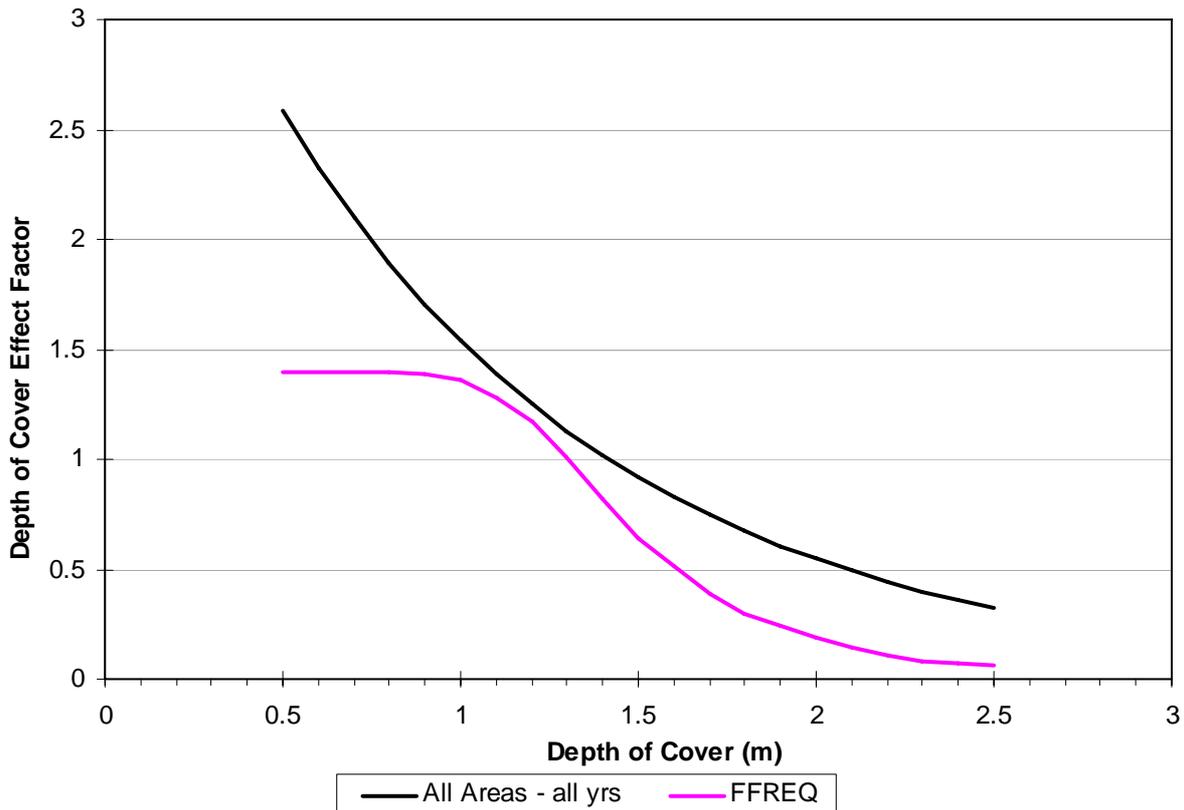


Figure 5: Derived depth of cover factor curve compared to UKOPA FFREQ

3.3 Comparison with IGEN/TD/2

In Figure 6, the derived depth of cover factor curve for all years of data is compared against the curve published in IGEN/TD/2 [1]. The curve in TD/2 is presented as normalised at the recommended minimum depth of cover of 1.1m. Therefore, in order to make a direct comparison, the depth of cover effect factor relationship derived in the current work has also been normalised to 1.1m following the procedure described in Section 2.5.

Note that, in order to apply this normalised version of the curve, the baseline hit rate should also be adjusted to correspond to a baseline depth of cover of 1.1m (in place of the mean value of 1.4m). In order to derive the same overall hit rate at a depth of cover of 1.1m, a factor of 1.39 (from the depth of cover factor curve in Figure 4) should be applied to the average hit rates. Hence, multiplying the baseline hit rates calculated for Rural and Suburban areas in Section 2.6 by a factor of 1.39, gives adjusted baseline hit rates for use with the relationship in Figure 6 of 5.17×10^{-4} per km.year for Rural area pipelines and 2.13×10^{-3} per km.year for Suburban pipelines.

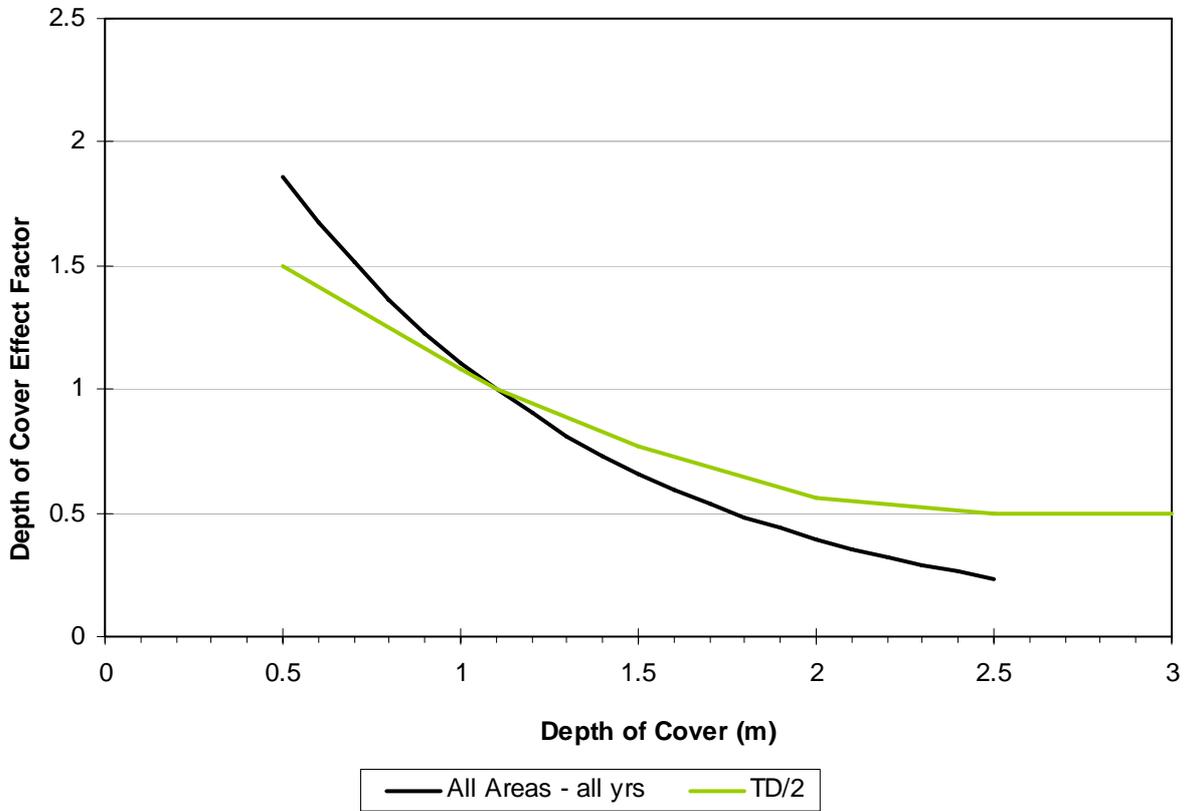


Figure 6: Derived depth of cover factor curve compared to TD/2 curve [1]

4 Conclusions

The purpose of the present study, undertaken for the UK Onshore Pipeline operators Association (UKOPA), was to derive a relationship between the hit rate and the depth of cover of pipelines in order that factors may be applied to the average hit rate (and hence the predicted failure frequency) to take account of the actual depth of cover in pipeline risk assessments (following the methodology described in IGEN/TD/2 and PD 8010 Part 3, for example).

The effect of depth of cover on the failure frequency of a pipeline has been studied in previous work (most recently reported in 2004), which derived relationships between the failure frequency for external interference and the depth of cover. The approach in all of these studies used the depth of cover measured in reported corrosion incidents to estimate the depth of cover distribution for all pipelines in the database (based on the assumption that corrosion is independent of depth of cover). The work reported herein updates and extends the 2004 study by applying the same approach to the analysis of data from the UKOPA database, which includes information from a wider range of UK Major Accident Hazard Pipelines (and not only natural gas transmission pipelines owned by Transco/National Grid as considered previously).

It is important to appreciate that the relationship derived is the variation in the hit rate with depth of cover and not a measure of the absolute value of the hit rate. For example, a direct comparison of the absolute hit rates would show a substantial difference between Rural and Suburban pipelines, because the historical hit rate for Suburban area pipelines is considerably higher than the hit rate for Rural area pipelines. For the UKOPA dataset analysed in the study, the hit rate for Rural pipelines was calculated to be 0.37 per 1000 km.years and 1.5 per 1000 km.years for Suburban pipelines (higher by a factor of approximately 4). The average depth of cover estimated for the pipelines in the dataset was approximately 1.4m (the difference in average depth of cover between Suburban and Rural pipelines was small and is neglected here).

The relationship derived is presented in Figure 7 below and shows the variation in hit rate with depth of cover (between 0.5m and 2.5m), where a factor of 1 corresponds to the average depth of cover estimated for the UKOPA pipelines. The "Depth of Cover Effect Factor" read from this figure corresponding to the actual depth of cover should be applied to the applicable hit rate in order to estimate the external interference hit rate for a particular pipeline.

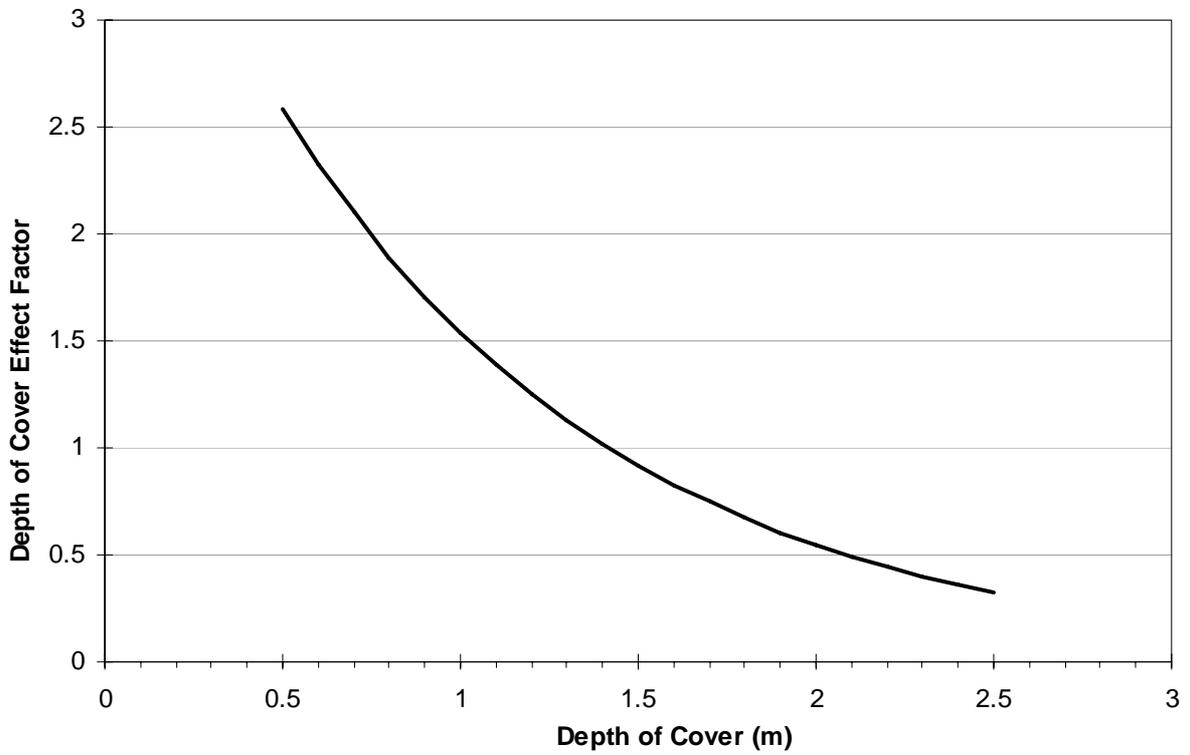


Figure 7: Depth of cover factors

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1. The Institution of Gas Engineers, IGEM/TD/2, 'Application of pipeline risk assessment to proposed developments in the vicinity of high pressure Natural Gas pipelines.', Communication 1737
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3. I. Corder and G. D. Fearnough, "Predictions of Pipeline Failure Frequencies". 2nd International Conference on Pipes, Pipelines and Pressure Systems, Utrecht June 1987
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5. G. Toes and C. Fowler, "Review Of Failure Frequency Predictions Used In Transmission Pipeline Risk Assessment", GL (formerly Advantica) Report R8010, April 2004
6. Pipeline Product Loss Incidents (1962 – 2010), Draft 7th Report of the UKOPA Fault Database Management Group, October 2011
7. UKOPA database 1962 – 2010, extract received from Rod McConnell on 1 November 2011, dated September 2011
8. E. Jager, R. Kuik, G. Stallenberg and J. Zanting "The Influence of Land Use and Depth of Cover on the Failure Rate of Gas Transmission Pipelines", Proceedings of 4th International Pipeline Conference, Calgary, IPC2002-27158

Appendix A Sensitivity Studies

A number of sensitivity studies were carried out to investigate the sensitivity of the depth of cover effect factors to the selection of the dataset used and the method of grouping data for statistical analysis.

The two datasets compared were the full UKOPA dataset covering the period 1962 – 2010 and a subset of this data covering the previous 20 years only (1990 – 2010). The two methods of grouping data compared were 0.2m depth of cover bands covering the range 0.5 to 2.5m depth of cover (“Case 1”) and a smaller number of bands, adjusted to give a more uniform spread of data by using broader depth of cover bands at the extremes of the distribution where data is sparse (“Case 2”). The depth of cover bands used in the two cases are given in Table 2 below.

Table 2 Depth of cover bands

ID	Depth of cover bands (m)			
	Case 1		Case 2	
	Min	Max	Min	Max
1	0.5	< 0.7	0.5	< 0.8
2	0.7	< 0.9	0.8	< 1.1
3	0.9	< 1.1	1.1	< 1.3
4	1.1	< 1.3	1.3	< 1.5
5	1.3	< 1.5	1.5	< 1.7
6	1.5	< 1.7	1.7	< 2.0
7	1.7	< 1.9	2.0	< 2.5
8	1.9	< 2.1		
9	2.1	< 2.3		
10	2.3	< 2.5		

The resulting four datasets were each analysed in turn to derive a best fit to the data following the methods described in Section 2, as follows:

- Case 1 (1962 – 2010)
- Case 1 (1990 – 2010)
- Case 2 (1962 – 2010)
- Case 2 (1990 – 2010)

The resulting plots of the natural log of the ratio of third party hit rates to corrosion incidents as a function of depth of cover are presented below in Figures 8 to 11 respectively, together with the equation of the fit to the data and the R² value (a measure of the goodness of fit).

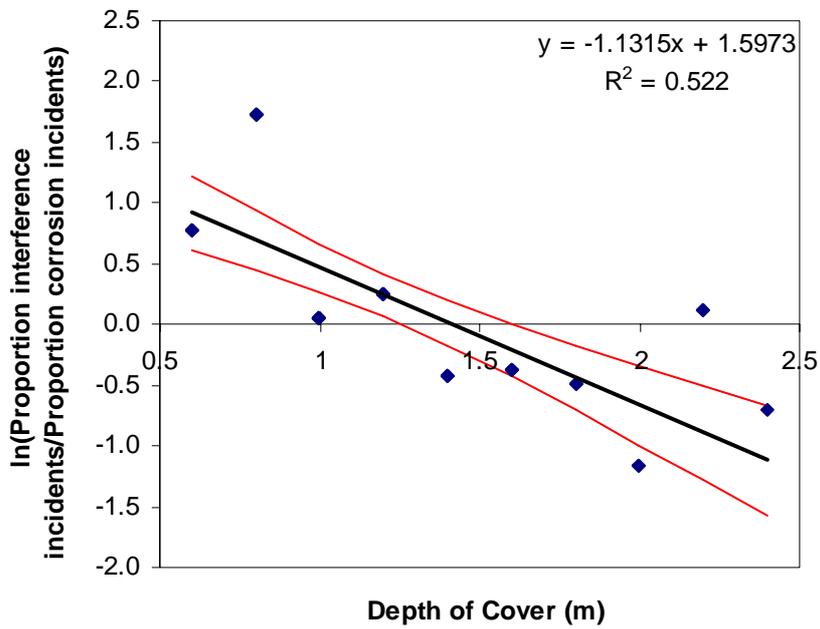


Figure 8: Case 1 (1962 – 2010)

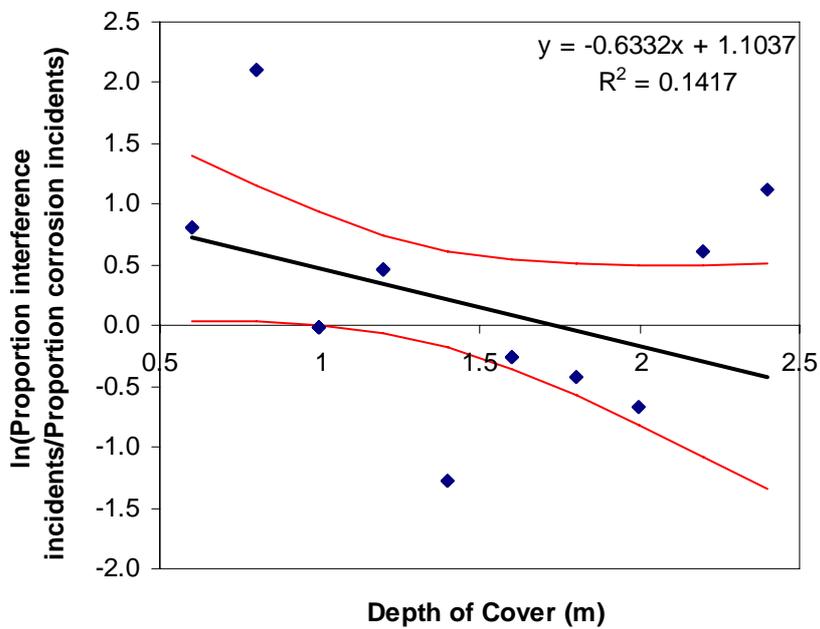


Figure 9: Case 1 (1990 – 2010)

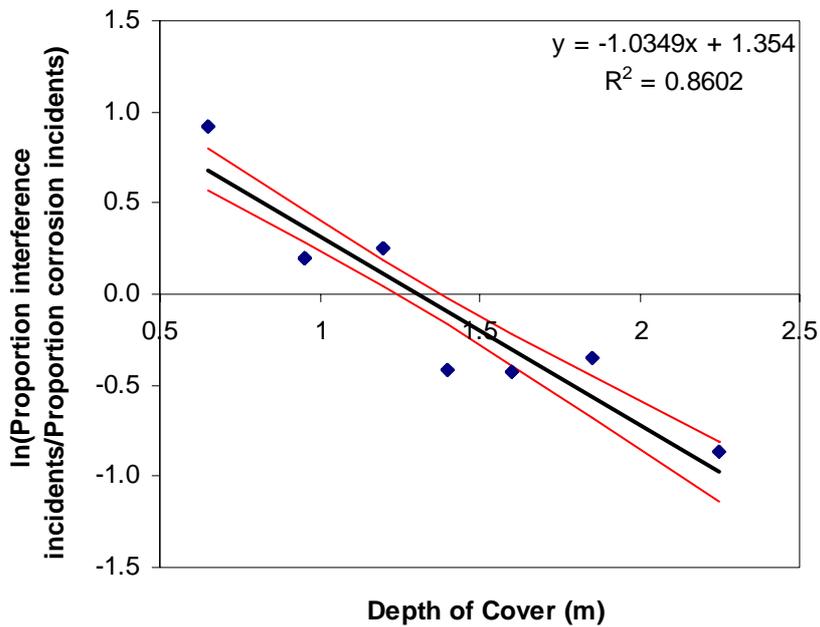


Figure 10: Case 2 (1962 – 2010)

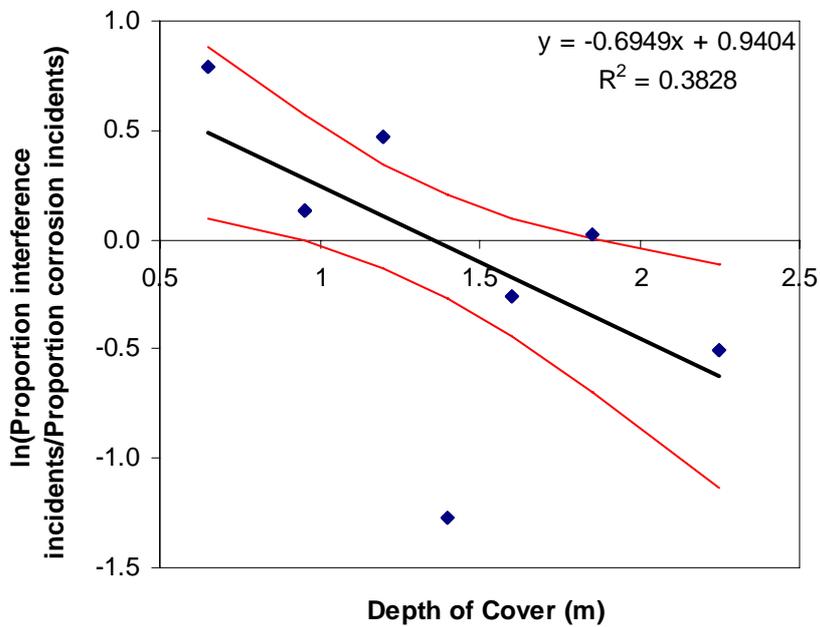


Figure 11: Case 2 (1990 – 2010)

Following the sensitivity analysis, Case 2 (1962 – 2010) was selected as the basis of the relationship described in the main report, as this represented the best fit to the data (i.e. the highest R^2 value).

Appendix B Statistical Method

In this work, linear fits have been carried out using the least squares method. The choice of a weight factor reduces the error made at the end points of a distribution, where the number of hits is relatively small. A weight factor is used, which is the rounded value of the square root of the total number of hits in each depth class. In other words: if y_i represents the hit frequency in the depth of cover class (band) with centre d_i , $i = 1, 2, \dots, n$, and n_i is the corresponding weight factor, then those a and b are to be found so that the function

$$F(a, b) = \sum_{i=1}^n n_i [\ln(y_i) - ax_i - b]^2$$

reaches a minimum.

The null hypothesis that the correlation coefficient is a random deviation from the zero correlation is tested according to R.A. Fisher by means of the t -distribution with $n-2$ degrees of freedom, where n is the number of data points (counted with multiplicity) used in the least squares method (so $n = n_1 + n_2 + \dots + n_r$):

$$t = |r| \cdot \sqrt{\frac{n-2}{1-r^2}}.$$

Here r is the square root of the coefficient of determination. If $t > t_{\alpha, n-2}$ with $\alpha = 0.05$, the hypothesis is rejected and we conclude that with 95% confidence there is indeed a linear correlation ($t_{\alpha, n-2}$ is the critical t value). Conversely if $t < t_{\alpha, n-2}$ then there is no linear correlation with the specified confidence.

The confidence limits are determined as follows: given the least squares regression line $y = ax + b$, estimated from data points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, the 95% confidence interval is given by

$$\left\{ ax_i + b \pm \sqrt{2F_{\alpha; 2, n-2}} s_e \sqrt{\frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_j (x_j - \bar{x})^2}} \right\}_{i=1, 2, \dots, n} \quad \text{with } s_e^2 = \frac{1}{n-2} \sum_j (y_j - ax_j - b)^2,$$

where $\alpha = 0.05$. Note that the confidence interval of the expected value corresponding with a fixed x_i is equal to

$$ax_i + b \pm t_{\alpha/2; n-2} s_e \sqrt{\frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_j (x_j - \bar{x})^2}}.$$

However, the latter interval does not take into account all combinations of values of a and b that lie within the joint confidence region for slope and intercept.